EYFS, KS1 and KS2 Calculation Policy

The National Curriculum 2014

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject.

The aims of this policy

Mastery is for all, and the aim of this policy is to ensure all children leave our school with a secure understanding of the four operations and can confidently use both written and mental calculation strategies in a range of contexts. It aims to ensure consistent strategies, models and images are used across the school to embed and deepen children's learning and understanding of mathematical concepts.

How should this policy be used?

This policy has been designed to support the teaching and planning of mathematics in our school. The policy only details the strategies, and teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum. The examples and illustrations are not exhaustive but provide and overall picture of what the mathematics in our school should look like. This is not a scheme of work and must be used in conjunction with our school maths policy and curriculum documents.

This policy sets out the progression of strategies and written methods which children will be taught as they develop in their understanding of the four operations. Strategies are set out in a Concrete, Pictorial, Abstract (CPA) approach to develop children's deep understanding and mastery of mathematical concepts. Children choose and use concrete objects to help them make sense of the concept or problem; this could be anything from real or plastic fruit, to straws, counters, cubes or bead strings. This is developed through the use of images, models and children's own pictorial representations (e.g. number lines, bar models, ten frame images) to support them to make links/ connect to the abstract mathematics using symbols. Children will move between concrete, pictorial and abstract representations again and again, often revisiting previous representations when a concept is extended. It is helpful to ask children questions such as , 'what does the 6 in this calculation represent?'/ 'what do the jumps on this number line represent?'.

Children should be encouraged to develop an efficient method of recording: this could be pictorial, tabular, jottings and eventually formal written methods. This will support them to access and make sense of the maths within a problem and convey and communicate their understanding to others. Even from the earliest years, children can be encouraged to become efficient, e.g., in y1 children can be asked to make their pictures of their 'number stories' simple, and understand that this is a 'Maths picture'. In y6 pupils will be encouraged to draw bar models to represent e.g. fraction and ratio problems.

Similarly, although formal calculations are taught in a progressive sequence according to the Curriculum 2014, all Maths learning is designed to equip children with a 'tool box' of skills and strategies that they can apply to solve problems in a range of contexts and understand the Maths behind the formal methods. So as a new strategy is taught it does not necessarily supercede the previous, but builds on prior learning to enable children to have a variety of tools to select from. As children become increasingly independent, they will be able to and must be encouraged to select those strategies which are most efficient for the task.

The strategies are separated into the 4 operations for ease of reference. However, it is intended that addition and subtraction, and multiplication and division will be taught together to ensure that children are making connections and seeing relationships in their mathematics. Therefore, some strategies will be taught simultaneously, for example, counting on (addition) and counting back (subtraction).

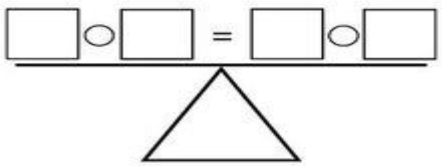
The Curriculum states that:

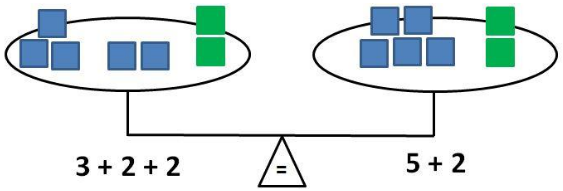
The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

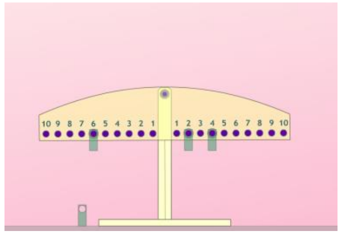
Therefore, some children will have extra support to access age related expectation; this could be: working with an adult in a smaller group, over-learning, pre-tutoring, or a modified activity which reinforces the same concept and addresses misconceptions. In addition, children who grasp the concept more quickly will be moved on to enriching activities and tasks, rather than accelerating into the next year's content.

Teaching equality

It is important that when teaching the 4 operations that equality (=) is also taught appropriately. Misconceptions that = means that children must 'do something' and that it indicates that an answer is needed are common and must be addressed early on. Teachers should present children with abstract calculations/ equations (e.g. 3 = 10 - 7) and problems which place the = sign in different positions, different context and include missing box problems. For example, ?+4=7; 7=3+?; <, >, or = 5+6__ 7+4. Equality can be demonstrated through the use of concrete objects; scales particularly provide a useful resource to demonstrate equality. Pictorial representations of equality can be used as shown below:







https://nrich.maths.org/content/id/5676/balancer.swf

Importance of vocabulary

The 2014 National Curriculum places great emphasis on the importance of pupils using the correct mathematical language as a central part of their learning. Children will be unable to articulate their mathematical reasoning if they lack the mathematical vocabulary required to do so. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers modelling and only accepting what is correct. For example:

✓	×
ones / units	not **just** 'units'
is equal to / equals	Makes
zero / nought	oh (the letter O)
number sentence/ calculation/ equation	sum/s

End of year expectations for calculations.

EYFS	Year 1	Year 2

• count reliably with numbers from one to 20.
• place numbers in order.
• say which number is one more or one less than a given number.
• using quantities and objects, they add two single-digit numbers and count on to find the answer.
• using quantities and objects, they subtract two single-digit numbers and count back to find the answer.
• solve problems, including doubling, halving and sharing.
	• read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
	• represent and use number bonds and related subtraction facts within 20
	• add and subtract one-digit and two-digit numbers to 20, including zero
	• solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? − 9.
	• solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
	• Find half of a quantity
		• solve problems with addition and subtraction:
		  ○ using concrete objects and pictorial representations, including those involving numbers, quantities and measures
		  ○ applying their increasing knowledge of mental and written methods
		• recall and use addition and subtraction facts to 20 fluently
		• derive and use related facts up to 100
		• add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
		  ○ a two-digit number and ones
		  ○ a two-digit number and tens
		  ○ two two-digit numbers
		  ○ adding three one-digit numbers
		• show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
		• recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
		• recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
		• calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
		• show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
		• solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts
		• find ½ ¼ and 2/4 and ¾ of a quantity

Progression in Calculations

Addition - overview of Foundation stage

Nursery
Before addition can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting, number order and number recognition through practical activities and games. Children also learn how to count using 1-1 correspondence (moving or pointing to each object as they count) and that anything can be counted, for example, claps, steps, jumps and sounds. This is reinforced by opportunities provided in both indoor and outdoor areas for the children to count e.g. counting building blocks, twigs etc. Children should also be encouraged to recognise small quantities without the need to count. This skill of subitising is a pre-requisite to calculation as well as helping children understand the quantity value of a number. The language of 'one more' helps with introducing addition.

Reception
Before addition can be introduced, children in Reception build on concepts taught in Nursery by working through the number objectives in the 40 – 60 month band of Development Matters. Children need to have a secure knowledge of number in order to begin addition. Children are then introduced to the concept of addition through practical games, activities and 'number stories'. These number stories need to include both structures of addition:

- **Aggregation:** this involves combining two existing groups, eg 5 red bears and 4 blue bears, *how many altogether*? This could be modelled by having two sets and physically combining them to make one group and counting all. This will *lead to* counting on from one number.

- **Augmentation:** this involves increasing the size of one set and asking '*how many now?*' Eg, 5 children in the park, 4 more come, how many now? This can be modelled by putting 5 small world play people in a 'park' and 4 more play people coming in. When asked 'how many now', children can first count all, and then be encouraged to move on to counting on from the first, known number. This will extend the understanding of addition beyond the idea of 'one more than 3 is 4'.

Subtraction

Nursery

In Nursery, children are to the concept of subtraction as ‘taking away’. Children must enact ‘number stories’ to understand the idea of finding out ‘how many are left?’. For example, 6 bears at the park, 2 go home, how many left at the park? The bears are moved, and sometimes hidden so that children can clearly see the ones which ‘are left’. Children start to understand that we can count backwards as well as forwards, and use number rhymes and songs which support this, eg, Five currant buns in a baker’s shop. The language of ‘one less’ helps with introducing subtraction.

Reception

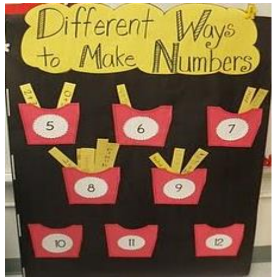
Children develop their understanding of subtraction using the following structures:

- **Taking away:** Through continuing to use ‘number stories’ to enact **‘taking away’** operations. Children act out subtractions to physically subtract a number of objects from a group.

- **Difference:** Children also need to experience subtraction as **‘difference’,** using the language of ‘which is more, 5 bears or 8 bears?’, and compare by lining up each set and seeing which is more. They also need to compare different sets of objects, eg, if there are 8 bears and 5 biscuits, have I got enough biscuits for each bear to have one? These objects can also be lined up and conversations about ‘how many more’ do I need/ have left over?’ need to be encouraged.

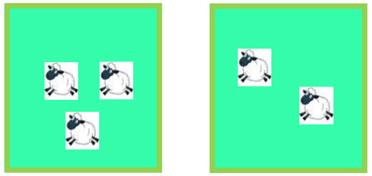
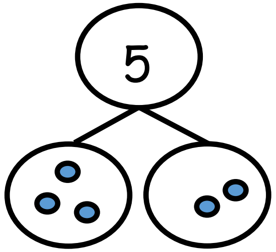
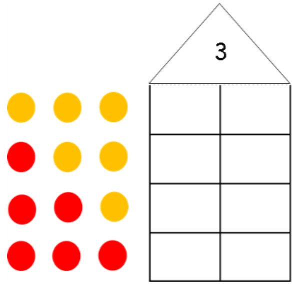
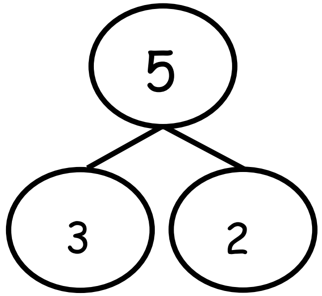
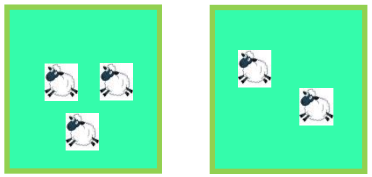
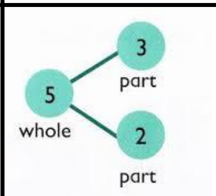
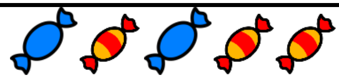
Recording in the Foundation Stage

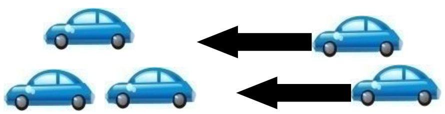
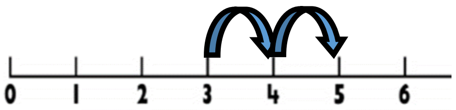
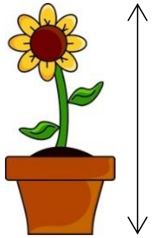
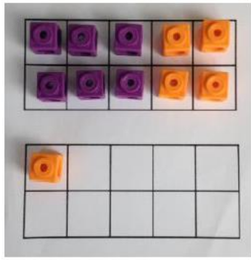
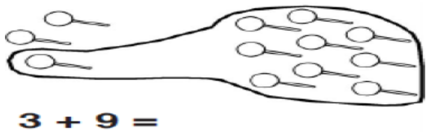
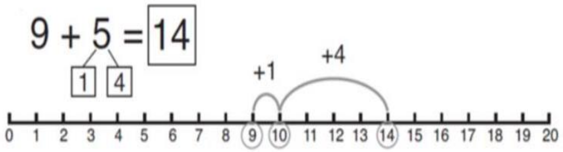
Children can ‘record’ their addition and subtraction number stories with their own picture, and start to make up and draw their own ‘stories’. At this stage, children will **NOT** be recording with calculations/ equation using abstract symbols, but can be encouraged to record with a number which is in their story.



Progression in Calculations

Addition/Subtraction: EYFS - Y2

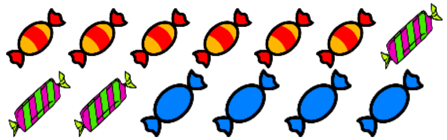
Strategies/ contexts	Concrete	Pictorial	Abstract
Partitioning and combining How many ways can a farmer put five sheep into two fields? Find as many ways as you can.		 	 0 + 3 = 3 1 + 2 = 3 2 + 1 = 3 3 + 0 = 3
Combining 2 groups to make a whole (aggregation) Counting sets of objects, combining then recounting using a 1:1 correspondence. There are three sheep in one field and two sheep in the other, how many altogether?	  	  	3+2=5 2+3=5 5 = 2 + 3 etc

Counting on (augmentation) Pupils should be taught to start at the biggest number and count on, using this as an opportunity to introduce the commutativity of addition. There are 3 cars in the car park, 2 more arrive, how many are there now? The flower is 6cm tall, it grows 2 cm in a week, how tall is it now? Jack is 7 years old, how old will he be in three years' time?	  	  	3 + 2 = 5 Make link to 'knowing this is right because we know that 5 can be 3 and 2.' Encourage recall of known number facts to develop fluency in mental calculations. .
Regrouping to make 10 To move on from the previous strategy, rather than counting on, children use their number bond knowledge and bridge to 10 e.g. if 4 + 6 = 10, so 4 + 7 must equal 11.	 6+5=11 Start with the bigger number and use the smaller number to make 10.	 	7p+4p=11p I have 7p, how much more do I need to make 10p. How much more do I add on now? If you know 10=7+3, what else do you know?

Adding 3 single digits

Use this method as an opportunity to develop fluent recall and application of known number facts including bonds and doubles.

Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



6 + 3 + 4 = 13

4 + 7 + 6 = 10 + 7

10

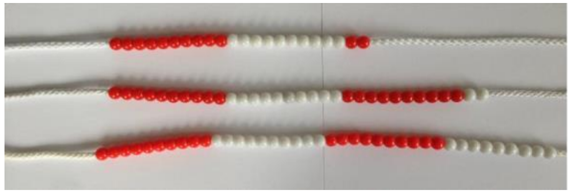
= 17

Partitioning to add

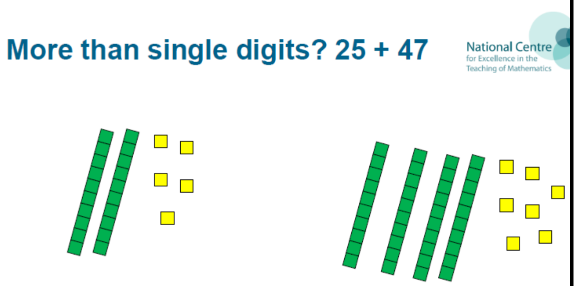
The emphasis in KS1 is to calculate using the value of the number i.e. knowing that 25 is 20 and 5 (as well as 2 tens and 5 ones)

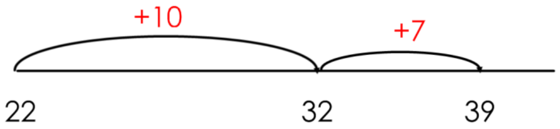
Informal recording is appropriate in KS1. This prepares children through a deep understanding of informal methods, for recording addition and subtraction formally in columns IN YEAR 3 .

Partitioning could be used to add two 2 digit numbers or to add a 2 digit number and a single digit number which involves crossing a tens boundary.

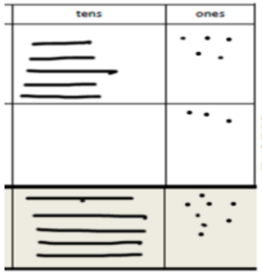


22+17=39

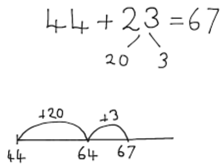




22+17=39



Begin with jumps of tens and ones then progress to jumps of all the tens and all the ones as children become more fluent.

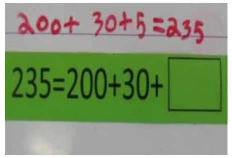


22+17

20 +10 =30

2+7=9

30+9=39

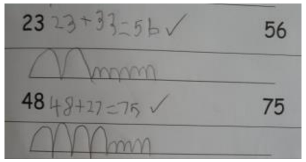


18 + 5

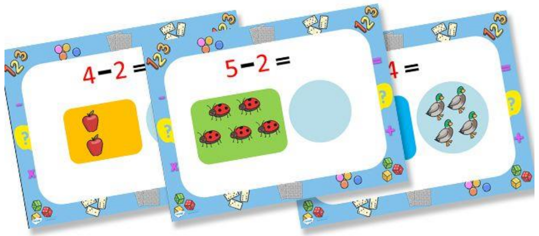
18 + 2 + 3

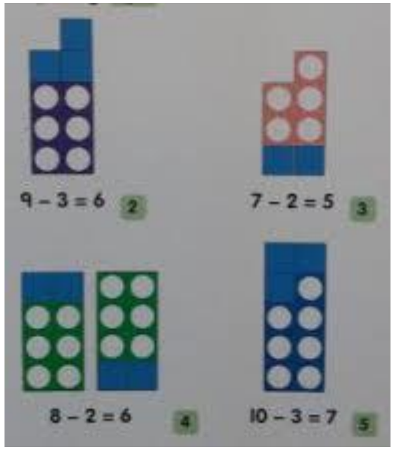
20 +3=23

Informal recording



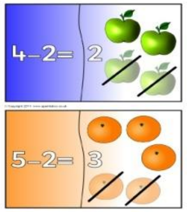
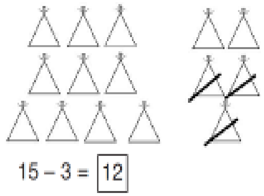
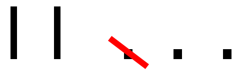
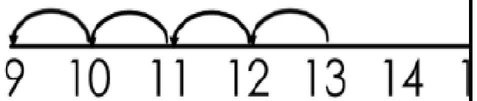
Counting on in tens and ones to solve missing number problems

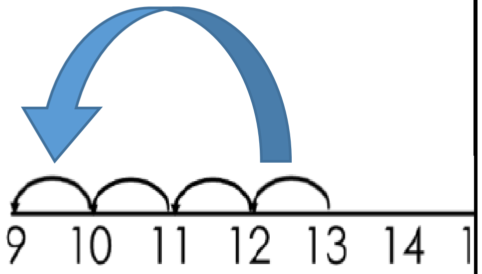
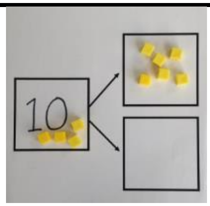
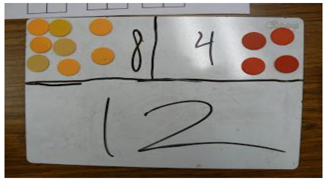
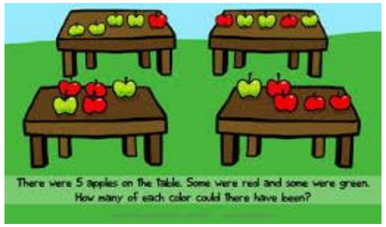
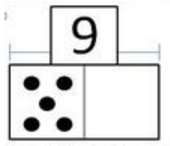
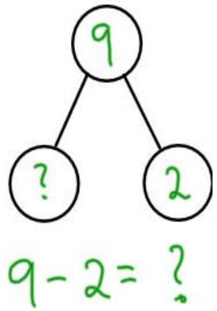
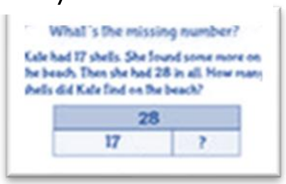

4−2 =
5−2 =
4 =

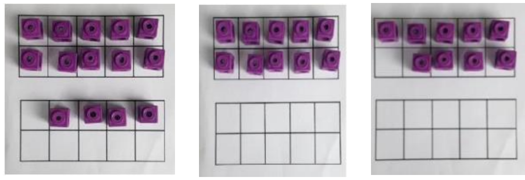
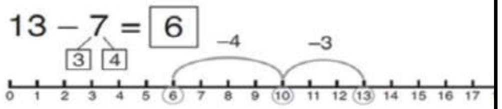
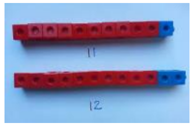
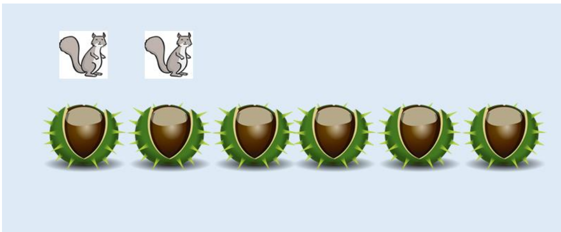
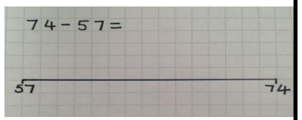
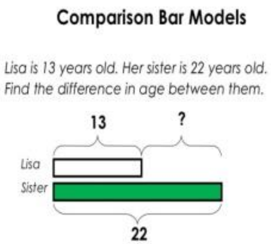

9 − 3 = 6
2
7 − 2 = 5
3
8 − 2 = 6
4
10 − 3 = 7
5

Progression in Calculations

Subtraction

Strategies	Concrete	Pictorial	Abstract
Taking away ones Use physical objects to demonstrate how something can be taken away. Move on to crossing out drawn representations. This can be developed by representing a group of ten with a line and ones with dots.	  	   23 - 1 = 12	18 -3= 15 8 – 2 = 6 There are 15 cakes in the shop. One cake is eaten, how many are left.
Counting back This strategy is useful only for knowing one or two less than a given number- it does not help them to see the relationships between addition and subtraction. Subtraction can be usefully represented on a number line, as the children can see the jumps." This is where I started, and this is where I have ended up"	 Use counters or objects and move only one or two away from the group as they are counted.	 13-4=9 This is a representation of subtraction, which leads on to more efficient jumping back. This should move on to 13-4 can be shown as	Put 17 in your head, count back 5. What number are you at? Use your fingers to help. 13-4= 13-3-1=9

		 	
Part, part, whole model This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.	 If 10 is the whole and 6 is one of the parts. What is the other part?  	  Children should be taught the skills to approach problems in a systematic way.	 I made 9 buns for the cake sale and I only had 2 left at the end. How many did I sell? 

Make it to the nearest 10 Use this strategy to subtract a single digit number from a 2-digit number. Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.	 14-5=9 Make 14 on the ten frame or with different coloured cubes to represent the ten and the ones. Take away the four first to make the nearest 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.		15 – 7= How many do we subtract to reach the next 10? How many do we have left to subtract?
Find the difference Children need to understand that difference involves comparing 2 sets of objects. Pupils should develop a good understanding of the meaning of 'difference', exploring the inverse relationship with addition by counting back and counting up. Children need to be encouraged to reason about the differences between numbers using known facts e.g." the difference between 9 and 15 is 6 because…".	 Practical resources to visualise 'difference' and recognise inverse relationships e.g. 12-1=11 and 11+1=12   	Use a blank number line to count up between 2 numbers to find the difference.   This is an image/ model which could be used towards the end of year 2, after the representations showing single units.	Lexie has 5 more strawberries than Jake. Jake has 11 cherries. How many does Lexie have? Look at the graph. Fewer children have green eyes than blue. What is the difference? 5+ ? = 13 ? + 5 = 13 13 = 5 + ? 15- ? = 9

Multiplication

Nursery and Reception

By the end of Reception, children are expected to understand the concept of doubling and to be able to double a number up to 10. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out 'doubling' by physically adding two equal groups together to find out the 'doubles' answer. They should be encouraged to recognise a doubles shown on fingers as a special pattern and learnt to subitise the total.

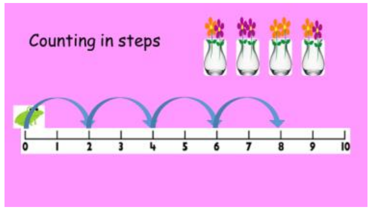
Children are then introduced to the concept of halving and sharing through practical games and activities. They act out 'halving and sharing' through activities such as sharing food for their Teddy Bear's Picnic, sharing resources equally to play a game. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.



Key Stage 1

Counting in multiples and times tables

In Year 1, children develop their counting knowledge and skills so that they can count in 'steps' of two, five and ten. They understand that this can make counting more efficient, giving the **same** total as if they count in ones. To become fluent in counting in these multiples, they should practice regularly using a variety of models and images to support them; e.g. **pairs** of items (socks, shoes) to count in twos, hands to help count in fives, and structured beads (on bead bar) to count in tens. They should also see this as equal jumps along a number line, and look for the patterns that these multiples make on a 100 square. Singing songs which use these multiples can support learning. Using a counting stick or wheel can help reasoning as well as being able to count backwards as well as forwards: e.g. asking children 'which number is halfway between zero and fifty?' in a count of fives. Becoming fluent in counting in multiples, and understanding how these numbers are derived, will help them become confident in solving problems involving multiplication and division.



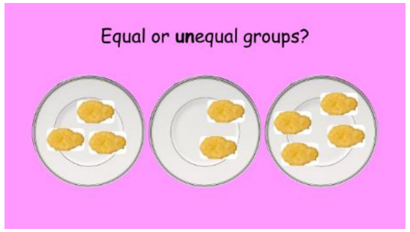
In Year 2 children consolidate their counting in twos, fives and tens, and learn to count in threes. They link the counting in multiples to times tables and should learn the times tables of two, five and ten to become fluent. They should be encouraged to use key facts to help them derive unknown facts, eg, if they know that five twos is 10, they can quickly work out six twos, without always having to start counting from 2.

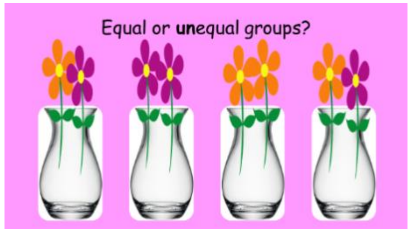
Solving problems using multiples: multiplication and division

Year 1
To help children understand the links between multiplication and division (both as grouping and sharing), it is essential that children understand what is meant by a 'group' and 'equal groups'. Understanding a collection of objects as one 'group' is called *unitising,* and many children will need a wide range of experiences and support to develop the understanding of this concept.

Working practically can help children understand the idea of a group; e.g. playing games in which they have to get into equal groups and those who are not in an equal group are 'out'; carrying out 'Show me' activities with counters or pictures, e.g. 'Tom put two apples in each bag, he had five bags,' (multiplication) or 'Dad made 12 cakes and put them into equal groups with three in each group' (division) or '8 apples shared between two plates'. Children can be encouraged to understand that counting in multiples helps them solve these problems.

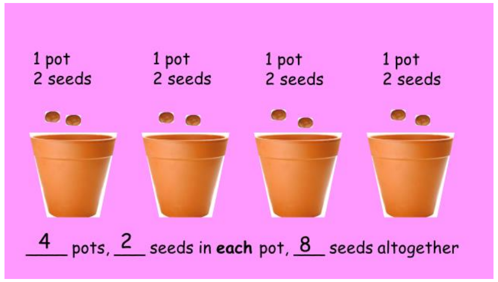
Looking at a range of real-life objects which are grouped can help consolidate the idea of 'equal groups', e.g. food packets, hands, wheels on a bike. Identifying equal and unequal groups can support this understanding, and children can be challenged to suggest *how* the groups could be made equal.





Children also need lots of practice to say:

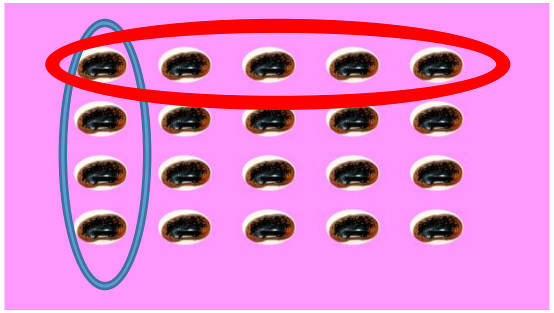
- how many **groups**
- how many in **each** group
- how many **altogether**



Children will find it helpful if they are sometimes encouraged to count the objects in a picture by using the number as an adjective, e.g. 1 pot, 2 pots, 3 pots, 4 pots; 2 seeds, 4 seeds, 6 seeds, 8 seeds, to help them answer questions about 'how many groups/ in each/ altogether'.

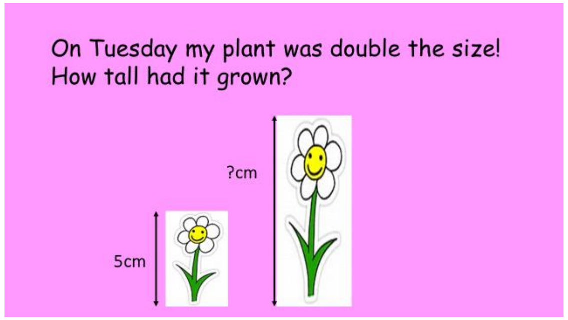
It is helpful if multiplication and division (both as grouping and sharing) is taught alongside each other and discussion about what the unknown information is, related to the above questions. Children will then start to see the relationship between multiplication and division.

Children can also be introduced to arrays and look for groups shown in arrays.



When solving multiplication and division problems, many children will be able to start to represent a problem using simple drawings which are meaningful to them.

Children should also be taught to solve doubling problems and some children in Year 1 will start to see the links between doubling and counting twos. Doubling introduces children to the concept of *scaling* which will be useful for them in KS2.



Year 2
STAFF DISCUSSION ABOUT 2X4 OR 4X2 AND LINKS TO DIVISION/ ARRAYS AND MODIFY

In Year 2 children continue to consolidate their understanding of multiplication and division through solving practical problems which can be enacted using equipment, e.g cubes and counters, and represented pictorially with simple pictures. Children are introduced to the symbols: X and ÷ to represent multiplication and division problems. These can be linked to repeated addition:

2 + 2 + 2 + 2 = 8

4 X 2 = 8 ('four times two equals eight')

They will be able to solve division problems more easily if they are fluent in counting in multiples. Alongside these symbols they should be encouraged to use maths sentences:

3 X 5 = 15

__ groups of __ is __ altogether

15 ÷ 5 = 3

__ put into groups of __ is __ groups

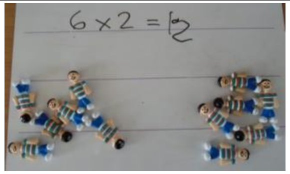
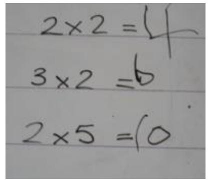
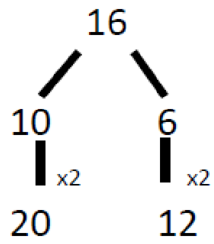
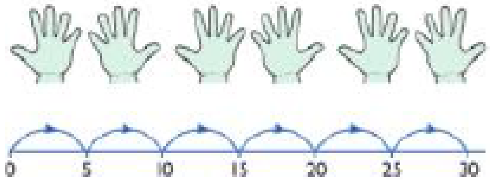
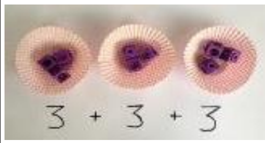
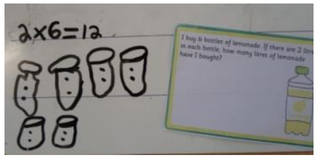
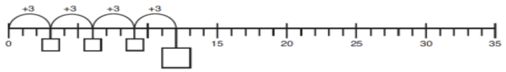
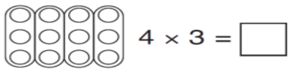
__ shared between __ is __ each

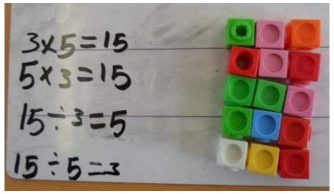
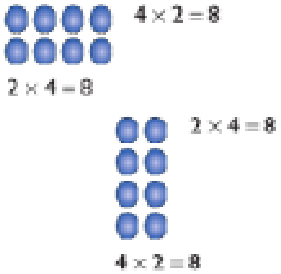
Children should be encouraged to see the links between multiplying by two and doubling, and dividing by two and halving.

They should explore how multiplication is commutative but division is not.

Progression in Calculations

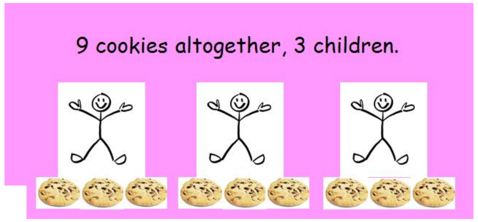
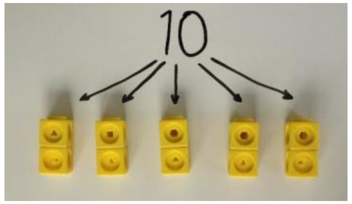
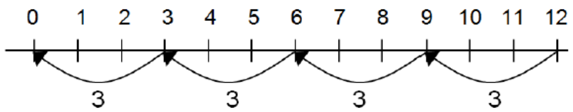
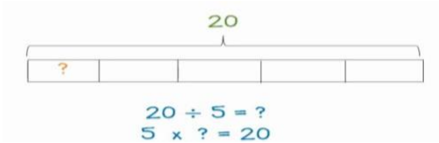
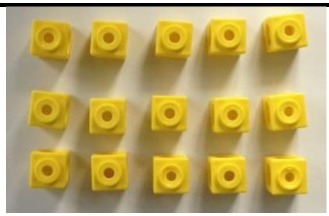
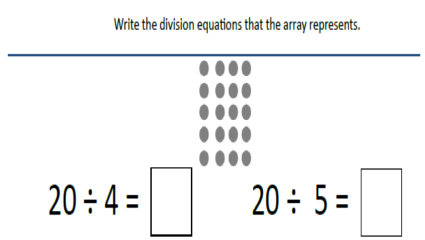
Multiplication

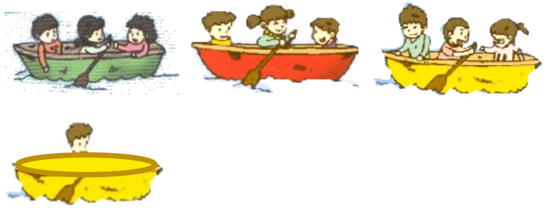
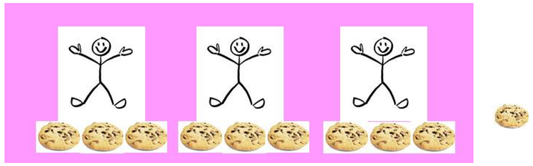
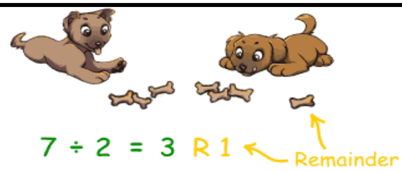
Strategies	Concrete	Pictorial	Abstract
Doubling Pupils should be encouraged to develop fluent mental recall of doubles and relate to the 2 x table.	  	Double 4 is 8 	  If I can see 10 wheels, how many bikes are there?
Counting in multiples Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'. Children should become increasingly fluent as they practise.	  	Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers and work out missing numbers in sequences both forward and backward. If I count in 2's will I get to the number 58?
Repeated addition Pupils should apply skip counting to help find the totals of repeated additions.	 5+5+5=15  3+3+3=9	 Pupils begin to recognise the relationship between repeated addition and multiplication.  	Write addition or multiplication sentences to describe objects and pictures.  2+2+2+2+2=10 2x5=10

Arrays showing commutative multiplication Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	 	Draw arrays in different rotations to find **commutative** multiplication sentences.  	3 children go to the park to hunt for pine cones. They find 5 each, how many do they find altogether? 5+5+5=15 3x5=15 5 children find 3 pine cones each. How many is that altogether? 3+3+3+3+3=15 5x3=15 Missing number problems which could arise from this are helpful. 8 X ? = 40 ? X 5 = 40 8 X 5 = ? Fluency will enable quick answers, rather than using the inverse.

Progression in Calculations

Division

Strategies	Concrete	Pictorial	Abstract
Sharing Here, division is shown as sharing. E.g. If we have 24 squares of chocolate and we share them between 3 people, each person will have 8 squares each.	 		Share 9 cookies between three people. 9 ÷ 3 = 3 Can you make up your own 'sharing' story and record a matching equation?
Division as grouping Here, division is shown as grouping. If we have ten cubes and put them into groups of two, there are 5 groups. This is a good opportunity to demonstrate and reinforce the inverse relationship with multiplication.	 Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Show jumps in groups. The number of jumps equals the number of groups.   Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group? Max is filling party bags with sweets. He has 20 sweets altogether and decides to put 5 in every bag. How many bags can he fill?
Division within arrays Use arrays of concrete manipulatives and images of familiar objects to find division equations. Begin to use dot arrays to develop a more abstract concept of division.		 Children can draw lines to divide their array	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Division with a remainder This strategy provides an opportunity to reinforce prior learning of odd and even and 'multiples' when exploring how numbers can and cannot be divided into different whole numbers.	10 children, 3 in each boat, how many boats do you need?  10 cookies, shared between 3 people, how many each? 	  	Complete written divisions and show the remainder using r. 

End of year expectations for calculations.

Year 3	Year 4
• add and subtract numbers mentally, including: • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds • add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction • recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods • find fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators • add and subtract fractions with the same denominator within one whole [for example, 1/5 + 2/5 = 3/5] • add and subtract amounts of money to give change, using both £ and p in practical contexts	• add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. • add and subtract fractions with the same denominator • solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.

Year 5	Year 6
• add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context (98 ÷ 4 = = 24 r 2 = 24 = 24.5 ≈ 25. 4 98 2 1) • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • use their knowledge of the order of operations to carry out

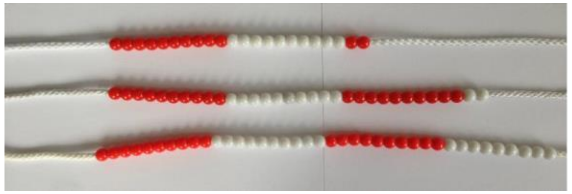
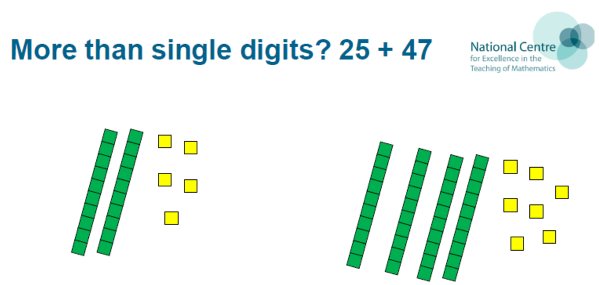
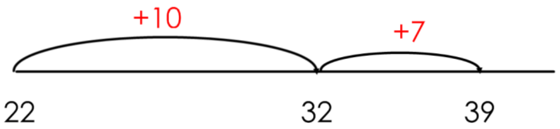
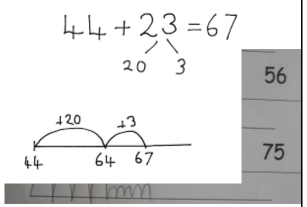
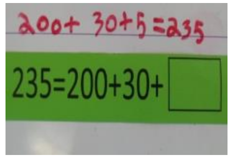
• add and subtract fractions with the same denominator and denominators that are multiples of the same number • multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	calculations involving the four operations • add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions • multiply simple pairs of proper fractions, writing the answer in its simplest form [for example,1/4 ×1/2 = 1/8] • divide proper fractions by whole numbers [for example,1/3 ÷ 2 = 1/6] • associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example,3/8] • multiply one-digit numbers with up to two decimal places by whole numbers • use written division methods in cases where the answer has up to two decimal places • solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts • solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison •

Progression in Calculations

Addition/Subtraction: Yr 3-6

In Key Stage Two, children are required to use facts learned in Key Stage One more quickly and have them 'at hand' in order to achieve fluency and be able to manipulate knowledge to apply to reasoning and problem solving in other areas . It is important that children 'know' these facts and are able to use the inverse; to know what numbers are and understand their relationships with other numbers. Applying the knowledge and fluency acquired in KS1 in areas such as percentages, measures, algebra and proportion work is now what the children need to be able to do. The 'fact fluency' is imperative so that the understanding, which the deeper and wider curriculum requires as they get older, has a firm foundation on which to ' build.'

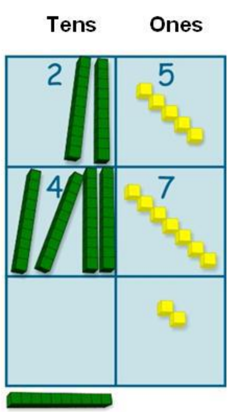
A deep understanding of place value is also necessary for children to be able to add and subtract larger numbers confidently. Children should be encouraged to discuss numbers both using their 'quantity value' (35 as thirty and five) and their column value (three tens and five ones) Teachers must also think about their choice of language when choosing a calculation method: number lines work with a quantity of a number, whereas column methods use their column value. Children must also be clear that when using number lines, only one number is partitioned, whereas column methods involve thinking about partitioning both addends.

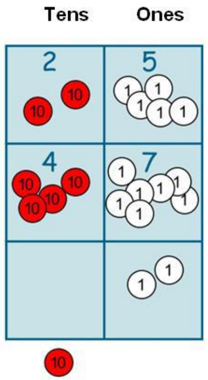
Strategies	Concrete	Pictorial	Abstract
Partitioning to add The emphasis in KS1 was to calculate using the value of the number i.e. knowing that 25 is 20 and 5 (as well as 2 tens and 5 ones) This will have prepared children for recording addition and subtraction formally in columns IN YEAR 3 . It is important that children re-visit familiar methods to link them to newer methods that involve columns. Partitioning could be used to add two 2 digit numbers or to add a 2 digit number and a single digit number which involves crossing a tens boundary.	  22+17=39 	 22+17=39  Begin with jumps of tens and ones then progress to jumps of all the tens and all the ones as children become more fluent. 	22+17 20 +10 =30 2+7=9 30+9=39  18 + 5 18 + 2 + 3 20 +3=23 Informal recording Counting on in tens and ones to solve missing number problems 236 + 167 13 90 300 393

Partitioning to add in columns. (Whole numbers and decimals including money)

There are 25 children on the play park. 47 more children join the, How many are there now? In the concrete example, the concept of carrying/ exchanging a ten into the column to the left as ten ones or a 'ten' is essential to move onto the abstract column method.

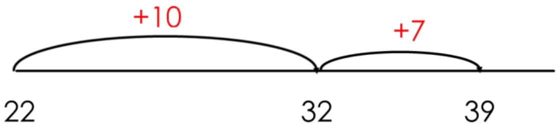
Or (visual example) 22 people are on the bus, 17 people get on. How many are there now? Children may use a blank number line to show their understanding of this problem still. The column method must be secure by the end of year 3, but mental methods should be used where possible.





In this example, exchanging ten ones for a ten is essential before the children move onto a completely abstract notation. The understanding of the value of numbers, the parts and the columnar system is imperative. Place value work forms a large part of the early Autumn term
Children understand that the exchanged ten is held below the line and should be added on to the other tens in the question.

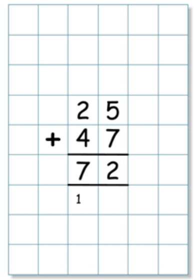
Children should have a secure understanding of adding using the quantity value of numbers before using a column method which uses the column value. The number line reinforces the quantity value as it deals with 17 as 'ten and seven'.



80+50 = 130
6 + 7 = 13
143

25 +47 =
5 +7 = 12 (10 and 2)
20 and 40 = 60
60 +12 = 72

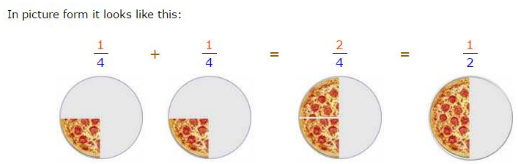
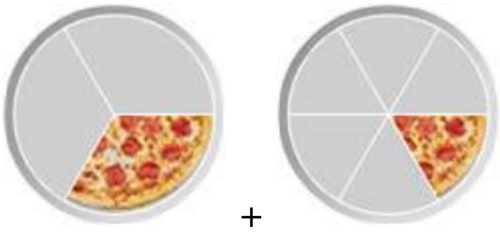
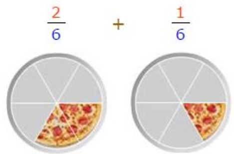
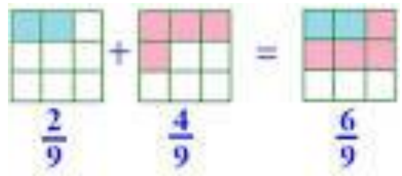
Here, the children are still dealing with the values of the numbers, and sung the partitioning method to add together numbers of tens and ones/ units.



Here, the children are using a purely abstract method which requires knowledge of the number system in columnar form.

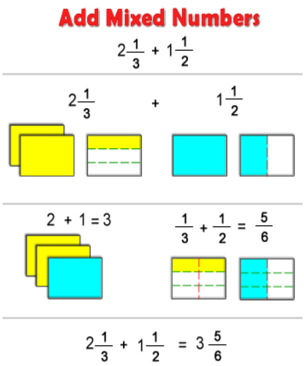
This extends in exactly the same way to knowledge and place value of decimals, where the children are able to further capitalise on the column/ place value of numbers to add only numbers of the same columnar value. Again, it requires much work before on what the

			numbers mean, and how big they are. The Dienes apparatus and images used in a different way allows the children to see the size of the scaling of the numbers relative to each column. 24.68 +17.94 24.68 +17.94 0.12 1.50 11.00 30.00 42.62 This extended method allows children to see what the numbers look like as they are added together as a column value before the use of exchanging/ carrying. 45.24 + 26.59 71.83 1 1

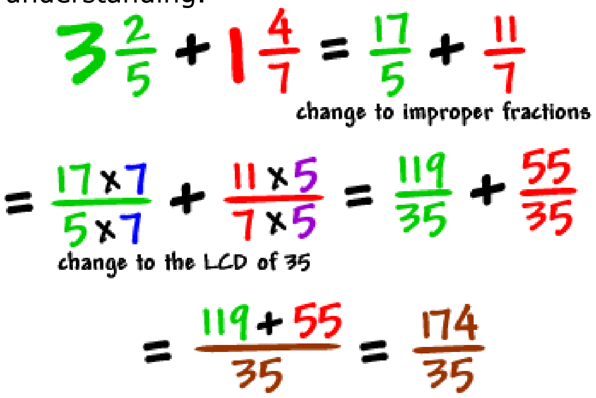
Adding fractions The children must be aware of the fact that the denominators must be the same, and you can only add two 'like' (same denominator) fractions together. This concept is understanding what fractions actually mean and is a prerequisite step. Eventually, children will be required to add 'unlike' fractions together, which have different denominators. The children have to change the fractions to equivalents in order to add them. This is related to multiplication. Children spend time becoming fluent in finding equivalent fractions long before they are required to use this knowledge in their work to solve problems such as these.	  1/3 + 1/6 	To calculate 2/9 + 4/9 children represent this pictorially to help them reason that because the denominator is the same, they can calculate by adding 2 and 4. 	$\frac{2}{9} + \frac{4}{9} = \frac{6}{9}$ 1/3 + 1/6 = 2/6 + 1/6 = 5/6
Adding mixed numbers. Children need to be able to convert from fractions to mixed numbers as part of their multiplication work and equivalence of fractions. Once this is fluency is enjoyed, they may	Using paper pieces for the concrete example. Children will be required to add together mixed numbers with different denominators. E.g. If I have 2 1/3 pizzas and Sam has 1 ½ pizzas, how many pizzas do we have together to share amongst our friends? Here the children recognise that the LCD	Here we can use informal jottings to help us to understand the meaning of the number as a whole. Thre are times when the whole number is required to be converted to an improper fraction, (especially when subracting), but the childrne need to see that both ways will provide the same answer. This will deepen understanding of the way that the numbers	Eventually, the children can Manipulate the fractions to suit their purpose, whether they change all the mixed number to improper or use a (more efficient) method which when adding requires only the fraction to be converted to the LCD.

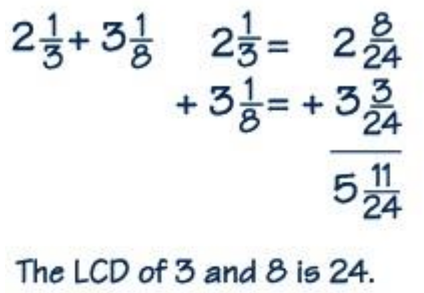
employ different strategies to add mixed numbers.
Children may either convert the complete mixed number to an improper fraction, or calculate with the whole numbers and convert the fraction to be of the same denominator. This will depend of course on the actual numbers involved and choices must be made to be efficient.

enables the children to add fractions more easily. For addding mixed numbers it is inefficient to change the mixed numebr into an improper fraction, and the wholes could be added as wholes before dealing with the fracion.



are behaving.
The colours help to keep the fractions clear at this informal jotting stage, to aid understanding.



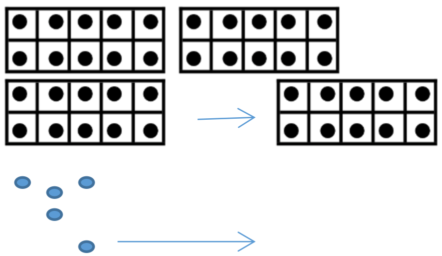


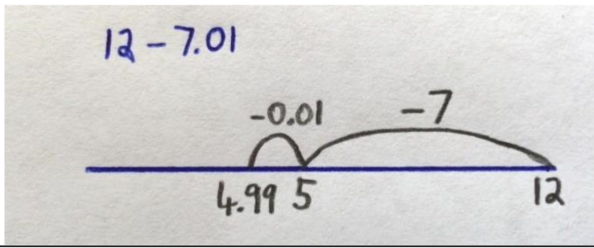
LCD= lowest common denominator.

Subtraction

Children should be working on subtraction alongside addition and know that one is the inverse of the other. Throughout KS2, children should be checking their calculations with the inverse calculation to achieve fluency and strengthen concepts. They should also at all times be looking for the most efficient method of calculation- mental, informal or written.

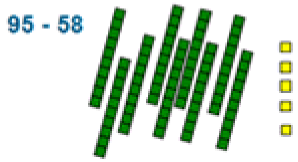
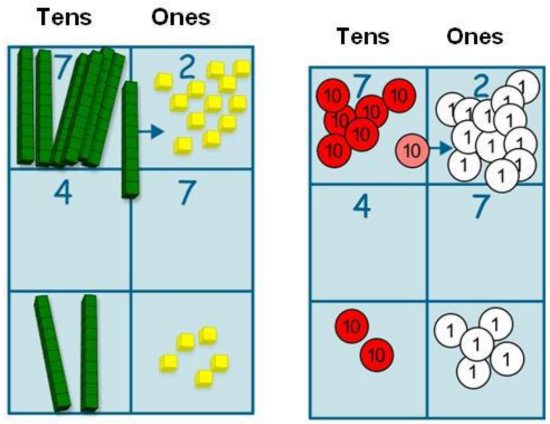
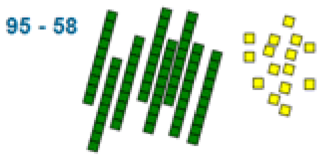
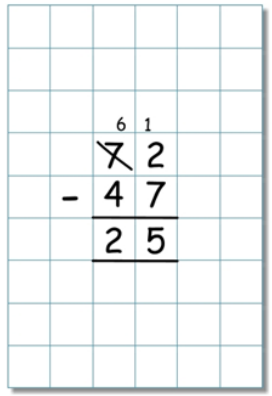
Partitioning , number value and place value.
Children need to recognise the numbers in parts, and recognise what the number is worth, so that they can efficiently jump back in hundreds, tens and ones/ units.

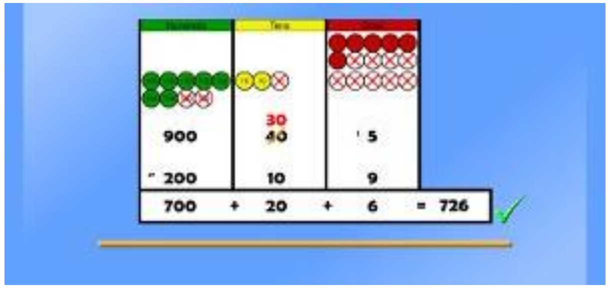
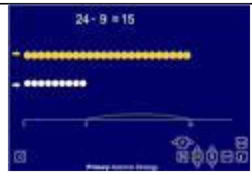
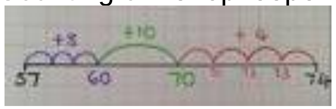
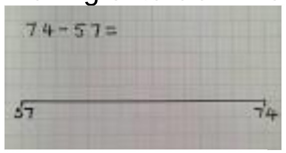
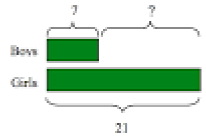


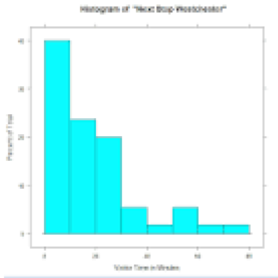
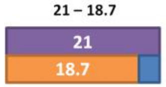
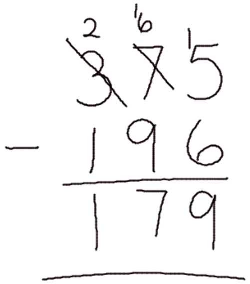
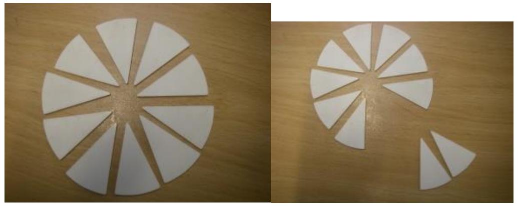


45-11 = 45-10 -1
= 35-1
= 34

Extending the idea to decimals
12- 7.01= 12-7 – 0.01

Exchanging The practicalities of column subtraction with a columnar value smaller than the corresponding number in the subtrahend, is a difficult concept, but with firm understanding of place value, this can be achieved with straws, dienes, counters etc.	   	90 5 80 15 -50 8 -50 8 72 = 60 +12 40 + 7 20 + 5	$^{8}9^{1}5$ 5 8 3 7 

	945-219 	After using the apparatus, children could be helped to move towards more abstract by using programs such as Number Gym-place value exercises.	945- 219 726 900-200=700 40---exchange ten for ones to leave 30 30-10 =20 5 ones (and the ten ones from the exchanged ones) =15 ones -9 = 6 Recombine 700 + 20 + 6
Difference Children see practically the difference between two quantities. They will use number lines, and then using the language-' what is the same and what is different?' they can see the link between difference and subtraction. From here on, they may use imagery in the form of bar model to aid the solving of problems, but the calculation will be a formal written column method.		 Counting on- shopkeeper's addition  Moving onto blank number lines  	24- 9 Inverse 74- 57 Eventually formal use of column subtraction can be applied to any situation involving subtraction or difference- but the concepts must be understood that though they use a similar written method, the idea is slightly different.

Solving problems may involve graph work/ measures and conversion or percentages.		  	 Decimals will be treated in a similar way as children become aware of parts of a whole one and have secure knowledge of place value. 21.00 18.70-
Subtraction of fractions. Similarly to addition, and at the same time as addition of fractions as the inverse, the idea will be that same denominators are mastered before equivalent fractions are needed for fractions of different denominators.	 $\frac{10}{10}$ - $\frac{2}{10}$   $\frac{5}{5}$ - $\frac{2}{10}$ = $\frac{10}{10}$ - $\frac{2}{10}$	Images can be drawn in any shape, not just circular to help with the concept of parts of a whole one.  $\frac{7}{9}$ - $\frac{4}{9}$ = ____  $\frac{4}{11}$ - $\frac{3}{11}$ = ____  $\frac{8}{9}$ - $\frac{4}{9}$ = ____	7/9-4/9= 3/9 $\frac{5}{6} - \frac{2}{15} = \frac{5 \times 5}{6 \times 5} - \frac{2 \times 2}{15 \times 2}$ $= \frac{25}{30} - \frac{4}{30}$ $= \frac{21}{30}$ $= \frac{7}{10}$

Accepted forms of calculation subtraction. These will be credited in exam situation.

789 + 642 becomes

```
   7 8 9
 + 6 4 2
 -------
 1 4 3 1
 -------
   1 1
```

Answer: 1431

874 – 523 becomes

```
   8 7 4
 – 5 2 3
 -------
   3 5 1
 -------
```

Answer: 351

932 – 457 becomes

```
   8  12 1
   9   3 2
 – 4   5 7
 ---------
   4   7 5
 ---------
```

Answer: 475

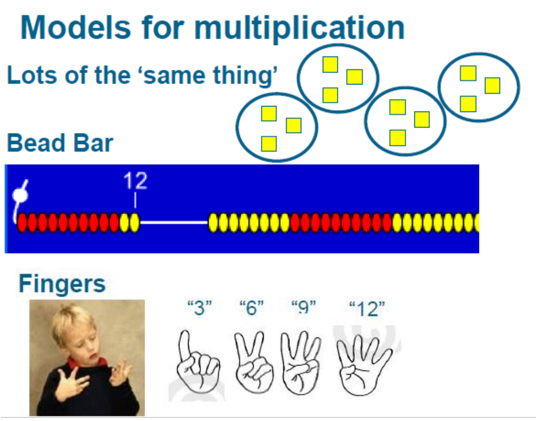
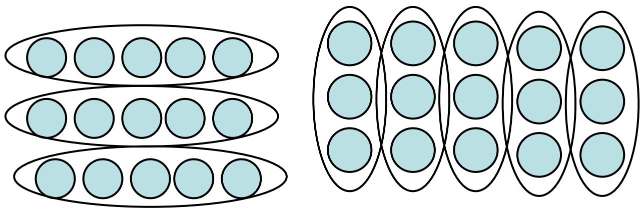
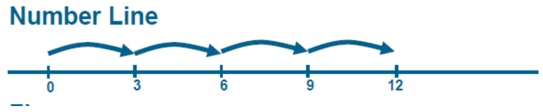
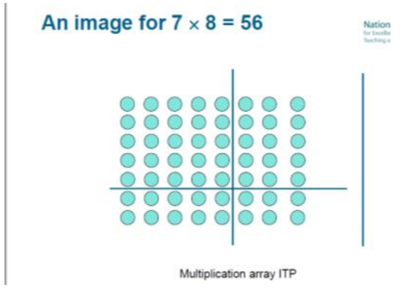
932 – 457 becomes

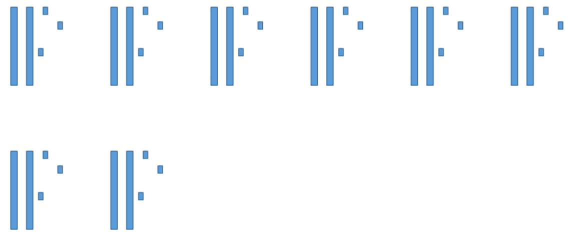
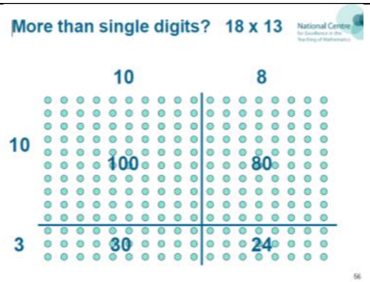
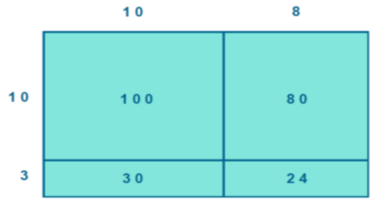
```
     1 1
   9 3 2
 – 4 5 7
   5 6
 -------
   4 7 5
 -------
```

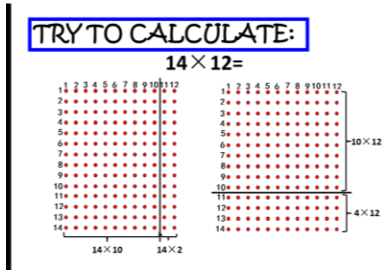
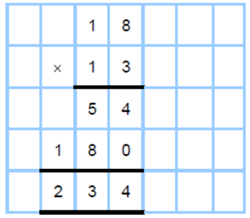
Answer: 475

Progression in Calculations

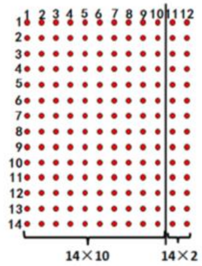
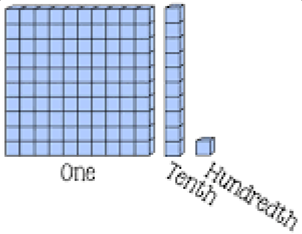
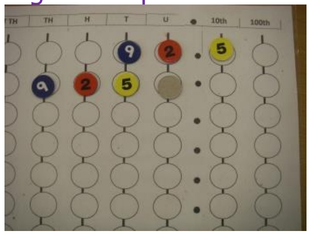
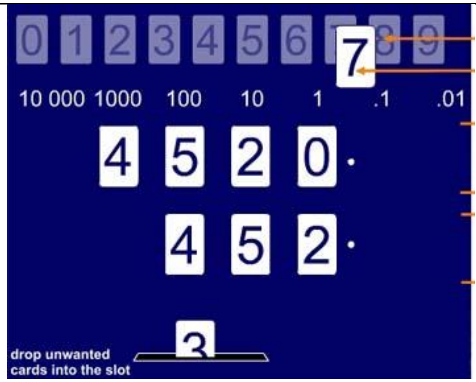
Multiplication and division: Yr 3-6

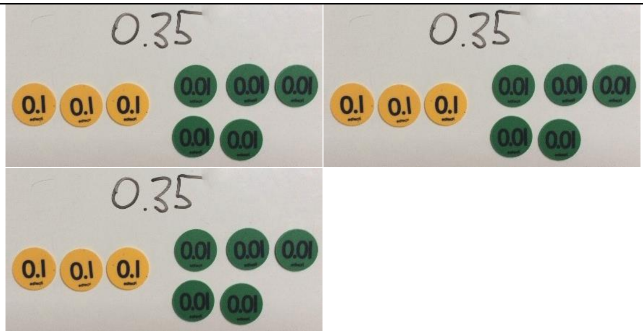
Strategies	Concrete	Pictorial	Abstract
Multiplication as ' lots of' or ' groups of' . It is imperative that the children learn tables- as multiplication facts and division facts, similar to learning addition and subtraction facts. These will become useful core basics, which are manipulated/ used to find other information and for reasoning. (especially fractions and division) Commutative 3 x 4 = 4 x 3 (any order) Distributive 3x8 = (2x8) + (1x8) Associative 30 x 6 = (3x10) x6	 	 3 lots of 5 5 lots of 3 	1x3=3 2x3=6 3x3=9 etc. 3x5 5x3

<table>
<tr>
<td>Multiplying two digits by one digit – informal.
This will help understanding of the place value, as well as the idea of multiplying all of the parts of the multiplicand by the multiplier.
There are 8 children , each pays £23 for a ticket to the zoo. What will the cost be altogether?</td>
<td></td>
<td><table><tr><th>x</th><th>20</th><th>3</th></tr><tr><td>8</td><td>160</td><td>24</td></tr></table>
Partitioning- informal method
38 x 5 = (30 x 5) + (8 x 5)
= 150 + 40
= 190</td>
<td>20x8=160
3x8= 24
184</td>
</tr>
<tr>
<td>Multiplying using a formal method .
Moving into a column formal method, the concept of partitioning is important as the children will methodically multiply each part of the multiplicand by the multiplier, and should know what each operation is achieving.
As the numbers get larger, the concrete will become less manageable.
At a concert, there are 13 rows of 18 chairs . How many chairs is this altogether?</td>
<td>
</td>
<td>

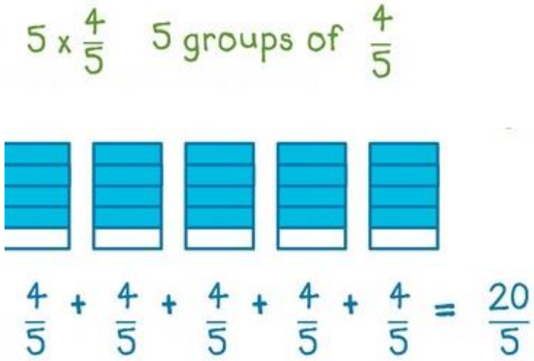
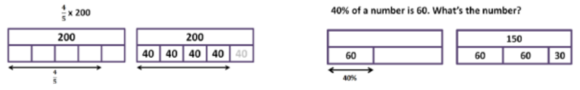

</td>
<td>

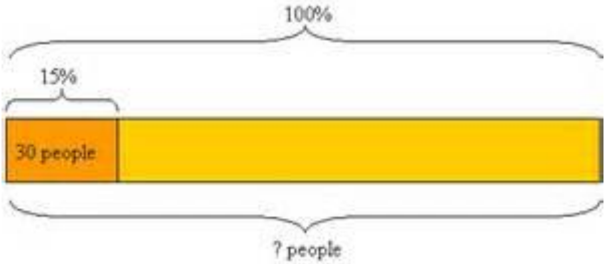
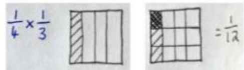
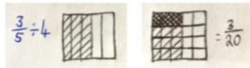
56
x 27
392 (7 x 56)
1120 (20 x 56)
1512
1
Here it is important for the children to see written down what each row represents.</td>
</tr>
</table>

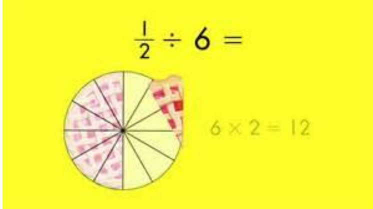
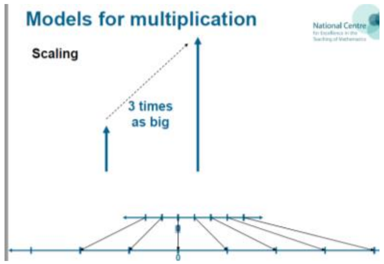
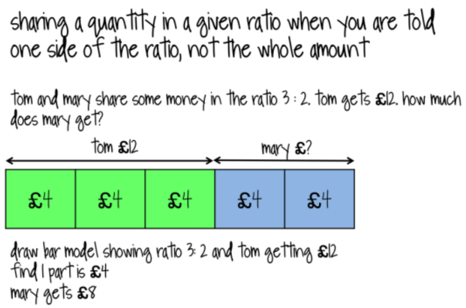
		14 × 12=  This image relates well to the informal methods (grid) which the children should be familiar with so far.	14 × 12= 14 × 12 28 ...(14)×(2) 140 ...(14)×(10) 168
Multiplying by powers of 10 Children are encouraged to think of numbers as part of the decimal system, where every number in a 'place' or column is ten times larger than the one to its right. This is particularly important in the introduction of decimals, so that the value of a tenth is seen as a very small part of a whole.	 **Examples of scaling.** Moving digits about on a place value mat, helps to reinforce the fact that all digits move, and none are 'left behind' It also reinforces 0 as a digit of importance.  Positional place value	 	4520 ÷10 = 452 203.03 x 100 = 20303 34 ÷1000 = 0.034

<table>
<tr>
<td>Multiplying by 3 and 4 digits.
Real life problems such as- If a field has dimensions 354m by 132 m, how much fertilizer would be needed at 100g per m²?</td>
<td>
Area of field- in metres square.
354 m wide, 132m long.</td>
<td>
<table>
<tr><th>x</th><th>300</th><th>50</th><th>4</th></tr>
<tr><td>100</td><td>30,000</td><td>5,000</td><td>400</td></tr>
<tr><td>30</td><td>9,000</td><td>1,500</td><td>120</td></tr>
<tr><td>2</td><td>600</td><td>100</td><td>8</td></tr>
</table>
30000
15000
+ 1700
20
8
46728
The informal jottings may be used to help with understanding, but at this stage the formal written method is the only method credited in exam situation.</td>
<td>354
132 x
708
10620
35400
46728
Formal written method.</td>
</tr>
<tr>
<td>Multiplying whole numbers by decimals.
Children should be encouraged to think about the place value, and calculate according to known methods, but reason about the size of the answer, including using estimation.
It costs £0.35 to buy a pen, how much would 3 pens be?</td>
<td>

3 x 0.35</td>
<td>Estimation would be very important, as then the decimal place is going to be in the right place. Reasoning about the size of the final answer would be important.
0.3x3=0.9
0.05 x 3 = 0.15
0.90 +
0.15 =
1.05
children are encouraged to line up digits correctly, and know that there are 0 in columns with nothing in.</td>
<td>0.35
3 x
1.05
1 1
" I know that if I had three lots of 0.3, the answer would be 0.9, so the final answer must be around that."
If the answer came out as £10.50, the child should realise that their decimal place must be wrong as this is nowhere near the estimate.</td>
</tr>
</table>

Multiplying Fractions and mixed numbers by a whole number. This could be taught through finding fractions of a number and showing the commutative aspect of the calculation. e.g. what is ½ of 46? What is 46 x ½ ? There are 30 biscuits in a pack. How many biscuits are there in 3 and 2/3 pack? There are 5 tanks of petrol. Each is 4/5 full. How many full tanks is this, and how much left over?	 The bottles are 4/5 full. How many full bottles can be made from the bottles in the cupboard?	How many 	$5 \times \frac{4}{5} = \frac{20}{5}$ = 4 full bottles- none left over.
Finding Fractions and percentages (associating fractions with division) Which is more, half of the big bar of chocolate or 2/3 of the smaller bar? (using area)		Bar model visualises finding fractions/ percentages of a quantity and finding the whole of a given percentage/ fraction. 	2/3 x area= $\frac{2 \text{ x area}}{3}$ OR (Area ÷ 3) x 2 Children need to see that the result is the same answer, and understand why.

A pair of trousers costs £30. What is the new price of the trousers in the sale? We can also find the whole number given a percentage. 15 % of a concert crowd do not turn up. That meant there were 30 empty seats. How many seats were there altogether?	Pizzas and cakes are the easiest things to share, but French sticks and rectangular shapes, including chocolate are a favourite to show that circles are not the only thing we can cut up!  		£30 ÷100 = 1% Or £30 ÷10 = 10% 10% of £30 = £3 30 % of £30= £3 x 3= £9 15%= 30 15%÷3 = 5% 30 ÷3= 10 5%=10 100% = 10 x 20= 200
Multiplying fractions by other fractions. 600 Seats at a concert are arranged into 4 blocks. One third of a block is sold to a school. How many seats did they have? I have 1/6 of a pizza, but can only manage half of that piece, what fraction of the whole pizza have I eaten?		 	¼ x 1/3 = $\frac{1\times1}{3\times4} = \frac{1}{12}$ 1/6 x ½= 1/12

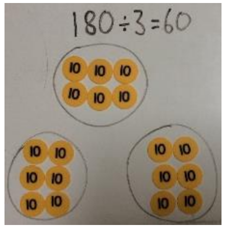
Dividing fractions by a whole number. Half a pizza is divided between 6 friends. What fraction of the original pizza do they get each?	  A 2Litre jug is half full. It is poured into 6 glasses. How much is in each glass?	 	$\frac{1}{2} \div \frac{6}{1}$ is the same as ½ x 1/6= 1/12 ½ ÷1/4= 1/8
Scaling including ratio and proportion Scaling is usually linked to measures, but is also linked to questions involving language such as 'four times as many' The plan of a garden is drawn to a scale of 1:20. What is the area of the garden?	e.g. Sam has 3 books, Lucy has 5 times as many. How many has Lucy?/ do they have altogether?   Children are required to measure accurately.	Ratio and proportion 	We encourage comparing sides and treating them as fractions/ proportional amounts and scaling up accordingly. 3:2 12 :? 3 x 4 =£12 so 2 x 4 =£8

Division including dividing where the answer has up to two decimal places.

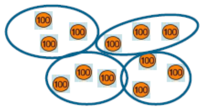
There are 180 children on three buses. How many children on each?

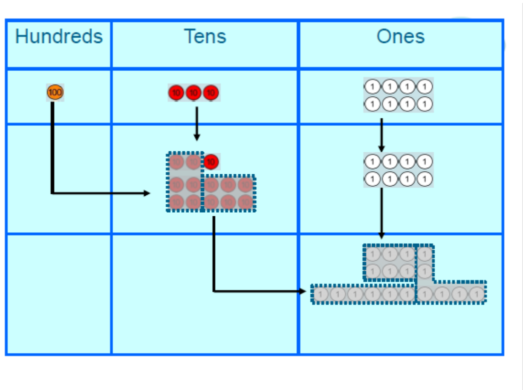
Sharing or grouping?

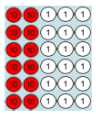
The children should know the difference between the styles of question. Although the answer and method may eventually become the same, understanding what you are actually doing is essential for problem solving and reasoning.

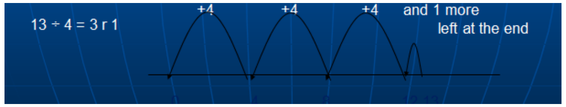


1200 ÷ 3



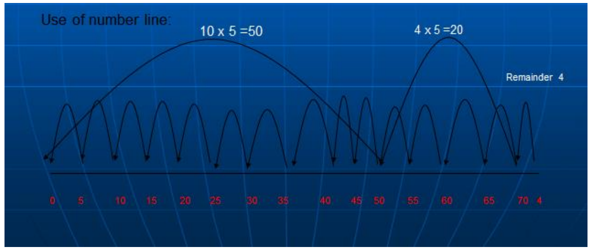






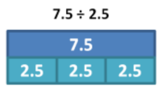
Counting forward and back on a number line

74 ÷ 5= 15 r 4



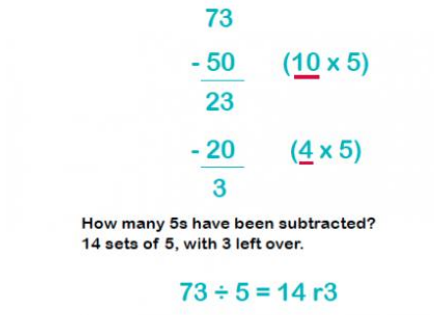
Jumping back on the number line in multiples of the divisor or 'Chunking'

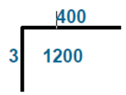
Bar model shows the relationship between the Dividend (7.5) and the divisor (2.5)



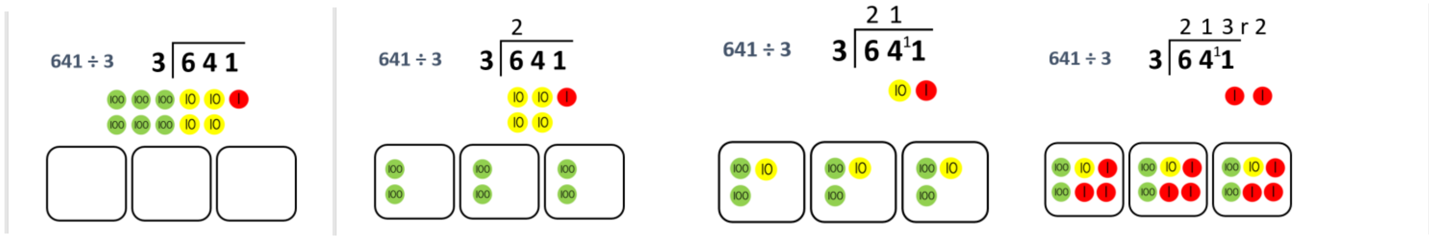
Leading to the multiples of the divisor method in vertical form.

73 ÷ 5=





Example of how ‘ bus stop’/ formal short division can be modelled.



Accepted forms of calculation multiplication and division- These will be credited with marks in an exam situation.

Short multiplication

24 × 6 becomes

```
    2 4
×     6
  1 4 4
    2
```

Answer: 144

342 × 7 becomes

```
    3 4 2
×       7
  2 3 9 4
    2 1
```

Answer: 2394

2741 × 6 becomes

```
    2 7 4 1
×         6
  1 6 4 4 6
    4 2
```

Answer: 16 446

Long multiplication

24 × 16 becomes

```
    2
    2 4
×   1 6
  2 4 0
  1 4 4
  3 8 4
```

Answer: 384

124 × 26 becomes

```
    1 2
    1 2 4
×     2 6
  2 4 8 0
    7 4 4
  3 2 2 4
  1 1
```

Answer: 3224

124 × 26 becomes

```
    1 2
    1 2 4
×     2 6
    7 4 4
  2 4 8 0
  3 2 2 4
  1 1
```

Answer: 3224

Short division

98 ÷ 7 becomes

```
     1  4
7 | 9 ²8
```

Answer: 14

432 ÷ 5 becomes

```
       8  6 r2
5 | 4 3 ³2
```

Answer: 86 remainder 2

496 ÷ 11 becomes

```
        4  5 r1
1 1 | 4 9 ⁵6
```

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

		2	8	r 12
1 5	4	3	2	
	3	0	0	
	1	3	2	
	1	2	0	
		1	2	

Answer: 28 remainder 12

432 ÷ 15 becomes

		2	8	
1 5	4	3	2	
	3	0	0	15×20
	1	3	2	
	1	2	0	15×8
		1	2	

$\frac{12}{15} = \frac{4}{5}$

Answer: $28 \frac{4}{5}$

432 ÷ 15 becomes

		2	8	·	8
1 5	4	3	2	·	0
	3	0			
	1	3	2		
	1	2	0		
		1	2		0
		1	2		0
					0

Answer: 28·8